

# Multi-echelon Inventory Model for Repairable Items Emergency

*By Agus Purnomo*

## Multi-echelon Inventory Model for Repairable Items Emergency with Lateral Transshipments In Retail Supply Chain

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**Abstract:** The theory on multi-echelon inventory systems is a core theory within supply chain management. Utilizing emergency lateral transshipments between retailers to meet customer demand can be an effective means for companies to improve service levels in a supply chain. This paper presents a Multi Echelon Inventory Model for Repairable Items Continuous Reviews using Emergency Lateral Transshipment. Based on case examples, obtained by the proportion of retail demand fulfillment immediately from stock on hand an average of more than 75%. Then the proportion of retail demand is met by emergency lateral transshipments average no more than 20%. Thus proved that the emergency lateral transshipments to increase the level of retail demand fulfillment. This can be seen on the results of calculations which was initially 75%, after using emergency lateral transshipments retail demand fulfillment rate rose by 20%.

**Key words:** Multi Echelon Inventory, Repairable Item, Emergency Lateral Transshipment Customer Service Level, Retail Supply Chain

### INTRODUCTION

The theory on multi-echelon inventory systems is a core theory within supply chain management, as it studies the optimal positioning of inventory in a supply chain. Quite some useful results are available on the optimal positioning of safety stocks, the effect of different types of lot-sizing rules, optimal control under capacity constraints, and the value of advance demand information (Van Houtum, 2005). The analysis of multi-echelon inventory systems that pervades the business world has a long history. Clark and Scarf (1960) introduced the concept of echelon stock. Inventory control in multi-echelon systems is known to be a challenging research area. Because of the complexity and intractability of the multi-echelon problem, Hadley and Whitin (1963) recommend the adoption of single location, single echelon models for the inventory systems. Sherbrooke (1968) constructed the METRIC model, which identifies the stock levels that minimize the expected number of backorders at the lower echelon subject to a budget constraint. This model is the first multi-echelon inventory model for managing the inventory of service parts. Thereafter, a large set of models that generally seek to identify optimal lot sizes and safety stocks in a multi-echelon framework were produced by many researchers (e.g., Deuermeyer and Schwarz, 1981; Moinzadeh and Lee, 1986; Axsäter, 1990; Svoronos and Zipkin, 1991; Nahmias and Smith, 1994; Aggarwal and Moinzadeh, 1994; Grahovac and Chakravarty, 2001). In addition to analytical models, simulation models have also been developed to capture the complex interactions of the multi-echelon inventory problems (e.g., Clark *et al.*, 1983; Pyke, 1990; Dada, 1992; Alfredsson and Verrijdt, 1999). The study of multi-channel supply chains in the direct versus retail environment has emerged only recently. The focus of this stream of literature is on channel competition and coordination issues in the setting where the upstream echelon is at once a supplier to and a competitor of the downstream echelon (e.g., Rhee and Park, 1999; Tsay and Agrawal, 2003; Chiang and Monahan, 2005).

While multi-echelon inventory control policies have been extensively studied, the theoretical basis for multi-echelon multi-channel inventory problem has not yet been well developed. One common arrangement of the extant multi-echelon inventory models assumes that the supply chain system consists of several locations whose supply-demand relationships form a hierarchy: each location places orders with one direct predecessor in the supply chain (see Svoronos and Zipkin, 1991 for more details). In this setting, one location may receive orders from several direct successors, but the successors have to be at the same echelon. Although a handful of papers have considered the model that allows some locations to bypass its direct predecessor and place orders with locations at a higher echelon or at the same echelon (e.g. Grahovac and Chakravarty, 2001), those

orders are considered as emergency orders and only happen in the event of a stockout at their direct predecessor. With the trend of adopting a multi-channel distribution strategy in the recent business environment and with the development of third-party logistics, including the emergence of highly competent suppliers such as Federal Express (Narus and Anderson, 1996), the inventory distribution system wherein one location can concurrently receive orders from more than one echelon is not uncommon. Nevertheless, the inventory modeling literature thus far offers little guidance in approaching this subject.

System of Multi-echelon inventory control is often used for extensive distribution system, means companies that can provide a high level of customer service, even in some cases the same goods produced in different factories to provide quick access to the market. For items that were damaged and can be repaired, then the supply of a product allows a system to hold a reserve item. Multi-echelon inventory system is divided into the lower echelon containing base-base as the first level of demand fulfillment services, and the two higher echelon with the warehouse as a second level of demand fulfillment services, Lee (1987) and Schwarz (1981). This paper discusses Multi-echelon model with a continuous review system for items that can be repaired with lateral transshipments in the retail supply chain. By using Multi-echelon inventory model with emergency lateral transshipment, it is expected to determine the proportion of demand compliance with the level of available inventory in the warehouse and at each base (retail) so that requests can all be met and the frequency of out of stock to a minimum. Another thing to be obtained from this research is to achieve the desired level of customer service or a certain service level set by the management company. This model is only used for system wide distribution and inventory items in question on this model are in independent circumstances, or not depending on other items, and inventory distribution management system is included as a pull system.

## MATERIALS AND METHODS

### **Research Design:**

#### **Terminology and Notation Used:**

In the discussion of models of Multi-echelon inventory control for repairable items with emergency lateral Transshipments will use terminology and notation as follows:

Terminology	Description
Emergency Lateral	Shipment or transfer from one base (retail) to the base (retail) others to meet demand when the retailer has no inventory. Emergency Lateral Transshipments only occur between the base at one polling the same group.
Backorder	A customer orders or commitments that are not filled or not fulfilled, because the inventory at the base (retail) is not sufficient to meet the demand for that item, so that the request be directed to the warehouse.
Inventory	Storage of goods, including raw materials, work in process (WIP), finished products, and supplies.
Item	Type or kind of a product.
Pooling Group	Regional distribution centers.
Base	Retail.
Central Depot	Warehouse as a distribution center.
Service Level	Policy management to customers relates to stockouts, as the ratio of the units that are supplied to the unit requested.

Notation	Description
$M$	The total number of base (retail) in all polling group.
$n$	Number pooling group.
$m_i$	Number of base (retail) in a pooling group $i$ . (Number of each base is assumed equal).
$T$	Transportation time from the warehouse to the respective base (retail).
$S_i$	Inventory levels at a base (retail) in a pooling group $i$ , for $i = 1, \dots, n$ .
$S_0$	The level of inventories in warehouses.
$D(t_1, t_2)$	The total demand of the entire base (retail) in the time interval between $t_1$ and $t_2$ , ( $t_1, t_2$ ).
$W_i(t)$	The number of orders that have not been fulfilled on a base (retail) in group $i$ in time $t$ , for $i = 1, \dots, n$ .
$W_i'(t)$	The number of orders that have not been fulfilled in the pooling group $i$ at time $t$ , for $i = 1, \dots, n$ .

$W_0(t)$	The number of orders that have not been fulfilled on the entire base at the time $t$ .
$B_0(t)$	The number of units in the warehouse backorder at the time $t$ .
$R(t)$	The number of units that have been repaired in the warehouse at the time $t$ .
$\mu$	The average number of items repaired in unit time.
$1/\mu$	The average repair time for damaged items (including transportation time the damaged items back to the warehouse).
$N_i$	The average number of Emergency Lateral Transshipment per unit of time in a pooling group $i$ , $i = 1, \dots, n$ .
$\lambda_i$	Average request arrival to each base at pooling group $i$ .
$\lambda$	Total average demand arrival at the base (retail).
$B_i(t)$	The total number of backorders at pooling group $i$ .
$\beta_i$	Proportion of demand that is filled with the arrival of stock on hand.
$a_i$	Proportion of demand that is filled with the arrival of emergency lateral transshipments.
$\theta_i$	The probability of arrival at pooling group $i$ demand that can not be met, so ask for a shipment from the warehouse.

**Model Description:**

As shown in Figure 1, this model has one central warehouse and M fruit base (retail). Then the base-base (retails) are grouped into  $n$  groups pooling off each other and every base in the pooling group is considered to be identical. Identical here means that the events that apply to a base (retail) is equal to the base (retail) other, so that sufficient attention in the discussion of one base model (retail) in a pooling groups.

The assumption used in this model will be explained as follows:

- Continuously supervise and supply warehouse inventory to each base / retail (continuous - review).  
Retails at the same polling group had an average improvement of identical arrival (from the warehouse to the base/retail).
- The demand of each base (retail) in a pooling group  $i$  follows a Poisson distribution with average arrival  $\lambda_i$ . In this case the arrival of a request to the warehouse.
- Transport time ( $T$ ) from the warehouse to the respective base (retail) is fixed and equal to the entire base (retail).  
Warehouse is a distribution center and as a place to repair a damaged item, and it is assumed there is no limit to the number of service, so there was no queue at the warehouse.
- It is assumed all the damaged items can be repaired.
- Demand that come the first time, will first be met (First Come First Serve)
- Demand are filled by lateral transshipment, would wait at the base until the arrival of units from the base (retail) are adjacent.
- When the warehouse supply inventory to the base (retail), and there is an item is damaged, then the damaged item is returned and repaired in the warehouse and was replaced by the existing inventory at the base (retail).
- If all base (retail) from a pooling group has no inventory, the demand will be immediately directed to the warehouse.  
Demand at a base (retail) is called a backorder if Emergency Lateral Transshipment as a way to meet the demand, or if all the base (retail) from a pooling group has no inventory and base (retail) are awaiting shipment from the warehouse.
- Genesis of the pooling group is considered synonymous with pooling the other group, meaning we only looked at one polling group that will represent the other group pooling.

**Algorithm Model:**

Processing steps performed in Multi-echelon inventory model for items that can be repaired with Emergency Lateral Transshipment is as follows:

- Determine the proportion of demand met by stock on hand ( $\beta_i$ ).

In determining the proportion of total demand will be met by inventories held by the base / retail (stock on hand), then steps should be done is as follows:

- Determining  $[R(t)]$  and  $[B(S_0)]$ .

To obtain the value of  $[R(t)]$  and  $[B(S_0)]$ , from Graves (1985), obtained by the following equation:

$$W_0(t+T) = B_0(t) + D(t, t+T), \text{ where} \quad (1)$$

$$B_0(t) = [R(t) - S_0]^+, \text{ and } [x]^+ \text{ show an max } [0, x] \tag{2}$$

$(W_0(t+T))$  is the amount of demand that is still waiting or have not met on the entire base, consisting of the number of back orders  $(B_0(t))$  in the warehouse and the amount of demand that has not been met from the base (retail) at an interval  $t$  and  $t + T$ , or  $D(t, t + T)$ . While the number of back orders is the difference between the number of units repaired  $[R(t)]$  with the level of inventory in the warehouse  $(S_0)$ .  $[R(t)]$  is the number of units repaired in the time interval  $t$ , here shows an experimental nature of the Poisson, where the chances of a successful time at specified intervals.

Palm Theorem (1938) in Graves (1985), states that:

"If the demand for an item is to follow a Poisson process with average demand  $m$  and time to repair for each damaged unit are independent and identical distribution on any distribution, with an average repair time  $T$ , then at steady state, the probability distribution of the number of units repaired  $R(t)$  Poisson distribution with mean  $mT$ ."

Based on Palm's theorem above, then  $R(t)$  Poisson distribution with mean  $l/m$ . So also with  $D(t,t+T)$  Poisson distribution with mean  $lT$ .

From equation (1) and (2), can be obtained distribution  $B_0(t)$  and distribution  $W_0(t+T)$  based on the value of  $R(t)$  as follows:

- If  $R(t) > S_0$ , then  $B_0(t)$  is positive it means going back orders. Since  $S_0$  is a constant, so that  $B_0(t)$  follows  $R(t)$  Poisson distribution with mean  $l/m$ , can be written with the equation:  $B_0(t) = \text{Poisson}(l/m)$
  - If  $R(t) < S_0$  then  $B_0$  is negative or 0 means there is no backorder, so the distribution  $W_0(t+T)$  is equal to the distribution  $D(t,t+T)$  is Poisson distribution with mean  $lT$ , can be written with the equation:  $W_0(t+T) = \text{Poisson}(lT)$
  - Conversely, if there backorder, distribution  $W_0(t+T)$  is the Poisson distribution, because the sum of 2 pieces of the Poisson distribution would produce a Poisson distribution. Then equation  $W_0(t+T)$  can be written:  $W_0(t+T) = \text{Poisson}(l/m + lT)$ . In practice, determining the distribution of the above  $W_0$  with less effective and difficult in the calculation, therefore used the results suggested in previous studies.
- b. Determining Value of Expectation and Variance of  $B(S_0)$  Based on equation (2), Graves (1985) showed that the expected value and variance of  $B(S_0)$  are as follows:

$$E[B(S_0)] = E[B(S_0 - 1)] - \Pr(R(t) \geq S_0) \tag{3}$$

where:

$$\Pr[R(t) \geq S_0] = 1 - \Pr[R(t) < S_0] \tag{4}$$

$$\text{Var}[B(S_0)] = \text{Var}[B(S_0 - 1)] - [E\{B(S_0)\} + E\{B(S_0 - 1)\}] \cdot [1 - \Pr[R \geq S_0]] \tag{5}$$

**C. Determining the Value of Expectation and Variance of  $(W'_i)$ :**

To determine the amount of  $b_i$  it is necessary to know the value  $W'_i$  large number of demand that is still waiting or have not been met in a pooling group  $i$ . During the arrival of demand to a Poisson distribution warehouse (with or without lateral transshipment), we can obtain the distribution of the amount of unmet demand in a pooling group  $i$  ( $W'_i$ ), by treating one group as a stocking polling location with inventories of mission, as suggested (Graves, 1985). In these circumstances (Graves, 1985), gives the probability distribution of  $W'_i$  as follows

$$\Pr[W'_i = k] = \sum_{x=k}^{\infty} \Pr[W_0 = x] \Pr[W'_i = k | W_0 = x] \tag{6}$$

Conditional distribution of  $\Pr[W'_i = k | W_0 = x]$  is the event that the number of orders that have not been fulfilled in a pooling group  $i$  ( $W'_i = k$ ), if the number of orders that have not been fulfilled on the entire base ( $W_0 = x$ ). From the description of the conditional distribution, as well as how to choose the  $k$  units in the pooling group  $i$  than  $x$  units with a probability of success for a pooling group  $i$  for  $m_i l_i / l$ . The probability of success is obtained based on the many demands on a group to a pooling- $i$  compared with the total demand that is  $m_i l_i / l$ . The phenomenon above is the events that followed the binomial distribution, because at each repetition, the odds of success are the same price that is  $m_i l_i / l$ . Consequently these equations can be written (given by Graves, 1985) as follows:

$$\Pr[W'_i = k] = \sum_{x=k}^{\infty} \binom{x}{k} (m_i \lambda_i / \lambda)^k (1 - m_i \lambda_i / \lambda)^{x-k} \Pr[W_0 = x] \quad (7)$$

Before continuing the discussion of the steps of the model used, Graves (1985) presents a method for distribution  $W'_i$  approach, mainly due to difficult and complex discussions when using equation [7] is raw. Graves (1985) using a negative binomial distribution model as an approximation of the distribution  $W'_i$  namely:

$$\Pr[W'_i = k] = \binom{r+k-1}{k} p^r (1-p)^k ; \text{ for } k = 0, 1, 2, \quad (8)$$

To obtain the expected value and variance of  $W'_i$ , Graves (1985) have shown that the expected value and variance of the number of orders that have not been fulfilled in a pooling group  $i$  ( $W'_i$ ) is given by:

$$E[W'_i] = \frac{m_i \lambda_i}{\lambda} E[B(S_0)] + m_i \lambda_i T \quad (9)$$

$$Var[W'_i] = \left(\frac{m_i \lambda_i}{\lambda}\right)^2 Var[B(S_0)] + \left(\frac{m_i \lambda_i}{\lambda}\right) \left(\frac{1 - m_i \lambda_i}{\lambda}\right) \quad (10)$$

$$E[B(S_0)] + m_i \lambda_i T$$

**D. Determining the Value of the Binomial Distribution Parameters  $R$  and  $p$ :**

To obtain the approximation of the distribution of  $W'_i$ , with a negative binomial distribution and the parameters  $r$  and  $p$ , the distribution parameters were obtained based on equation (11) and [12] are substituted into equation (9) and (10). Value of  $r$  and  $p$  is a constant parameter ( $0 < p < 1$ ) which satisfy:

$$E[W'_i] = \frac{r(1-p)}{p} \quad (11)$$

$$Var[W'_i] = \frac{r(1-p)}{p^2} \quad (12)$$

**e. Determine  $\Pr [W_i=k]$ :**

We consider a base in a pooling group  $i$ , with each base are identical and the inventory level  $S_i$ . when the base was not able to meet demand because of shortage of supply, the base will be filled with the request to another base (emergency lateral transshipment). In this case we will use the random rule in choosing a base that will make lateral transshipment, the odds of a selected base is  $1/m_i$ . Something similar is done to obtain equation (7) can be done to obtain the probability distribution of  $W_i$ . For any given value  $W'_i$ , the probability distribution in steady state conditions of  $W_i$  was written with the equation:



$$\Pr[W_i = k] = \sum_{x=k}^{\infty} \Pr[W_i' = x] \Pr[W_i = k | W_i' = x] \tag{13}$$

Conditional distribution of  $\Pr[W_i = k | W_i' = x]$  is the event that the amount of unmet demand at a base  $(W_i) = k$ , if the amount of unmet demand in a pooling group  $i (W_i') = x$ . From the description of the conditional distribution, as well as how to choose the  $k$  units in a base of  $x$  units with a probability of success on a base of  $1/m_i$ . The phenomenon above is the events that followed the binomial distribution, because at each repetition, the odds of success are the same price that is  $1/m_i$ . Consequently these equations can be written as follows:

$$\Pr[W_i = k] = \sum_{x=k}^{\infty} \binom{x}{k} (1/m_i)^k (1-1/m_i)^{x-k} \Pr[W_i' = x] \tag{14}$$

**f. Determining  $\beta_i$ .**

To get the magnitude  $\beta_i$ , namely the proportion or composition of orders which can be met based on the available inventory at base (stock on hand), by substituting equation (7) into equation (14). The equation for the value  $\beta_i$ , written as follows:

$$\beta_i = \sum_{k=0}^{S_i-1} \sum_{x=0}^{\infty} \binom{x}{k} (1/m_i)^k (1-1/m_i)^{x-k} \Pr[W_i' = x] \tag{15}$$

In this research, the value for good service levels ranged above 70% (depending on the company's management policy.) This value indicates kebaikkann of inventory model is used.

**2. Determining the total amount expected backorder at pooling group  $i E[B_i(t)]$ .**

Consider a situation many orders in a group to a pooling- $i$  greater than or equal to inventory in the pooling group, which  $W_i' \geq m_i S_i$  showed at least  $m_i S_i$  items still waiting or have not been met in a pooling group  $i$ .

With a probability of:

$$\theta_i = \Pr[W_i' \geq m_i S_i] \tag{16}$$

$$\begin{aligned} &= 1 - \Pr[W_i' < m_i S_i] \\ &= 1 - \Pr[W_i' = 0] + [\dots] + \Pr[W_i' = m_i S_i - 1] \end{aligned}$$

In this model we will use  $q_i$  as an approximation to the probability of arrival of requests at a pooling group  $i$  that can not be filled with stock owned (meaning the pooling group is out of stock), then so will be a backorder. However, these probabilities are not exact (actual), this is because when the number of orders on the pooling group  $i$  exceeds the supply ( $W_i' \geq m_i S_i$ ), in reality there is a possibility of a base with orders in excess of inventories at the base ( $W_i > S_i$ ), the base may be asked to base others who have the supply. This can happen because at the time of arrival of requests, there is the possibility of pooling the entire group does not have the inventory and the  $i$ -th base to backorder. In contrast, other base-base can be supplied from the warehouse, bringing the total number of items that are still waiting on a pooling group can be smaller than the  $m_i S_i$ . Thus the expectation of backorder at pooling group  $i$  are filled directly from the warehouse are :

$$E(W_i' - m_i S_i)^+ \tag{17}$$

In the previous explanation, it is known that the average amount of demand per unit of time is filled with the stock on hand at group  $i$ , namely  $m_i(\beta_i I_i)$ . Can then also determine the average amount of demand per unit of time in group  $i$  who became backorder due to demand be met directly from the warehouse, which is  $q_i(m_i I_i)$ . Thus, the average number of requests per unit of time that is filled with emergency lateral transshipment at pooling group  $i$  can be known, namely:

$$m_i \lambda_i - m_i(\beta_i I_i) - q_i(m_i \lambda_i) = m_i \lambda_i (1 - \theta_i - \beta_i).$$

Thus the arrival of demand met by emergency lateral transshipment is:

$$a_i = m_i \lambda_i (1 - \theta_i - \beta_i) / m_i \lambda_i = 1 - \theta_i - \beta_i \tag{18}$$

$a_i$  value obtained was used to determine the expected value of the number of emergency lateral transshipments per unit of time at pooling group  $i$ ,  $N_i$ , which is written by equation:

$$N_i = m_i \lambda_i \alpha_i \tag{19}$$

with Little's formula is known that the number of items in transit for emergency lateral transshipments is  $N_i T_i$ . Thus, the total number of back orders on the pooling group  $i$  [ $B_i(t)$ ], consisting of the number of back orders are filled directly from the warehouse, and the number of back orders are filled with emergency lateral transshipments. Written by equation:

$$E(B_i) = E(W_i - m_i S_i)^+ + N_i T_i \tag{20}$$

## RESULTS AND DISCUSSIONS

The following will be presented several examples of cases that are relevant to the characteristics of the model to measure the performance of the model, and will be solved by using algorithms that have been described earlier.

### **Input Data:**

The data used is a hypothetical data to illustrate the supply chain from retail, drawn from case studies using computer simulation results, the research conducted by Hau Lee (1987). Model problem for this case example, can be seen in Figure 2.

General data on sample cases will be outlined as follows:

1. Multi echelon inventory system has a distribution center and a pooling group ( $n = 1$ ).
2. In a pooling the group there are 3 retail, each of which are assumed identical and equal to ( $m_i = 3$ ).
3. Average repair time for damaged item (dimisalkan electronic goods) known ( $1/\mu = 0.5$ , then the number of items repaired in unit time ( $\mu = 2$ ).
4. It is known that the average total arrival demand is 13 times the average demand for each demand to a base ( $\lambda = 13 \lambda_i$ ).
5. Transport time ( $T$ ) = 2 days.
6. The average time for lateral transshipment ( $T_i$ ) = 1 (for all retail)

Other data such as average arrival demand ( $\lambda_i$ ), the level of inventory in the warehouse ( $S_0$ ), and the inventory level at the base at pooling group  $i$  ( $S_i$ ), are presented in Table 1 as follows:

### **Data Processing:**

Data processing is performed to find the proportion of the demand fulfillment in Multi-echelon inventory model as previously described in the algorithm model. Examples of cases described is divided into several cases with different characteristics by the addition of an average size of the arrival of demand at each retail ( $\lambda_i$ ), the level of inventory in the warehouse ( $S_0$ ), and the level of inventory at retail ( $S_i$ ).

### **Cases with $I_i = 0.04$ , $S_0=1$ , $S_i=1$ :**

1. Determine the proportion of demand met by stock on hand ( $\beta_i$ ).  
The result of the calculation are presented in Table 2 as follows:
2. *Expectations Determine Total Backorder  $E[B_i]$*



- a. Determine the proportion of demand met in warehouse( $q_i$ )  $q_i = 1 - (0.504)+(0.290)+(0.127)+(0.050)+(0.017) = 0.02$
- b. Determine the proportion of demand that is filled with *Emergency Lateral Transshipment* ( $a_i$ )  $a_i = 1 - 0.02 - 0.84 = 0.14$

then the amount of the expected value of the number of emergency lateral transshipments per unit of time at pooling group i is:

$$N_i = (3)(0.04)(0.14) = 0.016$$

So the total number of back orders is obtained by using equation (20), namely:

$$E(B_i) = [0.546-3]^+ + (0.016)(1) = 0.016$$

**Cases with  $l_i = 0.04$ ,  $S_0=1$ ,  $S_2=2$ :**

1. Determine the proportion of demand met by stock on hand ( $b_i$ ).  
The result of the calculation are presented in Table 3 as follows:

**2. Expectations Determine Total Backorder  $E\{B_i\}$ :**

- a. Determine the proportion of demand met in warehouse( $q_i$ )

$$q_i = 1-(0.504)+(0.29)+(0.127)+(0.05)+(0.017)+ (0.006)= 1 - (1.00)= 0.00$$

- b. Determine the proportion of demand that is filled with *Emergency Lateral Transshipment* ( $a_i$ )

$$a_i = 1 - 0.00 - 0.99 = 0.01$$

then the amount of the expected value of the number of emergency lateral transshipments per unit of time at pooling group i is:

$$N_2 = (3)(0.04)(0.01) = 0.0012$$

So the total number of back orders is obtained by using equation (20), namely:

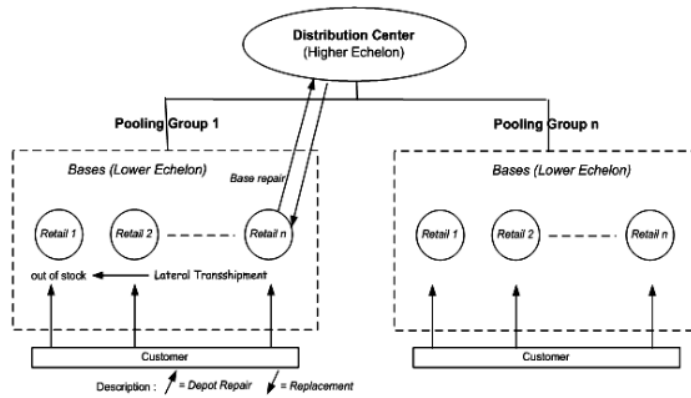
$$E(B_i) = [0.546-6]^+ + (0.0012)(1) = 0.0012$$

Calculation results next to  $b_i$  same as the examples above, are presented in Table 4. Calculation results next to the value of  $q_i$  and  $a_i$  same as the example above, can be seen in Table 5 and Total Proportion filled Demand Fulfillment with stock on hand ( $\beta_i$ ) and filled with Emergency Lateral Transshipments ( $a_i$ ), can be seen in Table 6. (The calculation is performed with the help of software support Delphi version 5)

**Analysis:**

Emergency Lateral Transshipments used in multi-echelon inventory system to improve service level costumers. Parameters that are important and will be analyzed as a measure of performance of the model: backorder level, because these parameters affect the percentage of service level. In this model, demand at a base is said to backorder, if:

- a. Emergency lateral transshipment as a way to meet demand. From the calculation, the proportion of demand fulfillment that is filled with lateral supply, not exceed an average of 19%, meaning that the value is quite influential on the level of demand fulfillment.
- b. All the base in a pooling group has no inventory and only waiting for shipment from the warehouse. From the calculation, the proportion of demand that can not be met by stock on hand and waiting for supplies from the warehouse ( $q_i$ ) is almost entirely near zero, meaning that almost all the demand can be met by stock on hand and with the way emergency lateral transshipments. Thus, the composition or the proportion of customer orders that are not filled or not met is very small, and these values show kebaikkan of multi-echelon model with lateral transshipments.



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Fig. 1: Multi-Echelon Inventory Model for Repairable Items with Emergency Lateral Transshipment.

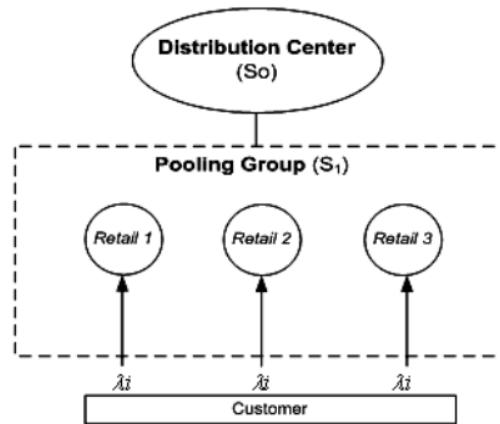


Fig. 2: Multi-Echelon Inventory Model for a sample case of the 3 retail supply chain.

Table 1: Average data arrival demand, inventories at the warehouse, and inventory at the retail.

$\lambda_i$	$S_o$	$S_i$	$\lambda_i$	$S_o$	$S_i$	
0.04	1	1	0.4	12	3	
		2			4	
	1	3				
0.06	2	2		0.6	30	4
		1				2
	2	3				
0.08	4	1	14		14	4
		2				4
	1	5				
0.1	6	1		21	21	6
		2				7
	2	4				
0.2	4	3	0.8		35	5
		1				3
	2	4				
0.2	6	1		20	20	5
		2				5
	3	6				

Table 1: Continue.

	3		7
	4		8
10	2	30	4
	3		5
	4		6
		40	7
			4
			5
			6

Table 2 : Calculation results of step ( $\beta_1$ ).

	Calculation	Value
a	$R(t)$	Poisson (2.08)
	$B(S_0 - 1)$	$R(t)$
b	Ekspektasi $B(S_0)$	1.326
	Variansi $B(S_0)$	1.256
c	Ekspektasi $W'_i$	0.546
	Variansi $W'_i$	0.76
d	$r$	2
	$\beta_1$	0.71
e	Pr [ $W'_i = 0$ ]	0.504
	Pr [ $W'_i = 1$ ]	0.29
	Pr [ $W'_i = 2$ ]	0.127
	Pr [ $W'_i = 3$ ]	0.05
	Pr [ $W'_i = 4$ ]	0.017
f	$\beta_1$	0.84

Table 3 : Calculation results of step ( $\beta_2$ ).

	Calculation	Value
a	$R(t)$	Poisson (2.08)
	$B(S_0)$	$R(t)$
b	Ekspektasi $B(S_0)$	1.326
	Variansi $B(S_0)$	1.256
c	Ekspektasi $W'_i$	0.546
	Variansi $W'_i$	0.76
d	$r$	2
	$\beta_2$	0.71
e	Pr [ $W'_i = 0$ ]	0.504
	Pr [ $W'_i = 1$ ]	0.29
	Pr [ $W'_i = 2$ ]	0.127
	Pr [ $W'_i = 3$ ]	0.05
	Pr [ $W'_i = 4$ ]	0.017
f	$\beta_2$	0.99

Table 4: Proportion of Demand Fulfillment filled with Stock on Hand ( $\beta_i$ )

Average demand arrival ( $\lambda_i$ )	Stocking level		Approximation	Average demand arrival ( $\lambda_i$ )	Stocking level		Approximation	
	$S_0$	$S_i$			$S_0$	$S_i$		
0.04	1	1	0.98	0.4	12	3	0.95	
		2	1			4	0.99	
	2	1	0.99		18	3	0.99	
0.06		2	1	0.6		4	1	
	2	1	0.96		30	2	0.97	
		2	1			3	1	
	4	1	0.98			4	1	
0.08		2	1	0.1	14	4	0.91	
	4	1	0.96			5	0.98	
		2	1			6	1	
	6	1	0.98			7	1	
0.1		2	1	0.1	21	4	0.97	
	4	1	0.92			5	0.99	
		2	1			6	1	
		3	0.99			35	3	0.98
	6	1	0.96			4	1	
	2	1			5	1		

**Table 4:** Continue.

3	1	0.8	20	5	0.92		
0.2	5	2	0.95			6	0.98
		3	1			7	1
		4	1			8	1
	10	2	0.99		30	4	0.9
		3	1			5	0.98
		4	1			6	0.99
						7	1
					40	4	0.98
						5	0.99
						6	1

**Table 5:** Demand Fulfillment Proportion who met at the warehouse ( $\theta_i$ ) and filled with Emergency Lateral Transshipments ( $\alpha_i$ )

Average demand arrival ( $\lambda_i$ )	Stocking level		Approximation		Average demand arrival ( $\lambda_i$ )	Stocking level		Approximation	
	$S_o$	$S_i$	$\theta_i$	$\alpha_i$		$S_o$	$S_i$	$\theta_i$	$\alpha_i$
0.04	1	1	0.02	0.14	0.4	12	3	0.05	0.14
		2	0	0.01			4	0	0
	2	1	0.01	0.11		18	3	0.01	0.09
		2	0	0.01			4	0	0.03
0.06	2	1	0.03	0.16	0.6	30	2	0.04	0.16
		2	0	0.02			3	0	0
	4	1	0.01	0.12			4	0	0.01
0.08	2	2	0	0.01		14	4	0.09	0.16
	4	1	0.04	0.16			5	0.01	0.1
		2	0	0.02			6	0	0.05
	6	1	0.02	0.14			7	0	0.02
0.1	2	2	0	0.01		21	4	0.03	0.12
	4	1	0.08	0.19			5	0	0.05
		2	0	0.04			6	0	0.02
0.2	3	0	0			35	3	0.02	0.12
	6	1	0.04	0.17			4	0	0.04
		2	0	0.03	0.8		5	0	0.01
		3	0	0			20	5	0.08
		5	2	0.05	0.15			6	0.01
		3	0	0.05			7	0	0.05
		4	0	0.01			8	0	0.02
	10	2	0.01	0.09		30	4	0.1	0.15
		3	0	0.02			5	0.02	0.1
		4	0	0			6	0	0.04
							7	0	0.02
						40	4	0.02	0.11
							5	0	0.04
							6	0	0.01

**Table 6:** Total Proportion filled Demand Fulfillment with stock on hand ( $\beta_i$ ) and filled with Emergency Lateral Transshipments ( $\alpha_i$ )

Average demand arrival ( $\lambda_i$ )	Stocking level		Approximation	Average demand	Stocking level		Approximation
	$S_o$	$S_i$			$\beta_i + \alpha_i$	$S_o$	
0.04	1	1	0.98	0.4	12	3	0.95
		2	1			4	0.99
	2	1	0.99		18	3	0.99
		2	1			4	1
0.06	2	1	0.96	0.6	30	2	0.97
		2	1			3	1
	4	1	0.98			4	1
0.08	2	2	1		14	4	0.91
	4	1	0.96			5	0.98
		2	1			6	1
	6	1	0.98			7	1
0.1	2	2	1		21	4	0.97
	4	1	0.92			5	0.99
		2	1			6	1
0.2	3	3	0.99		35	3	0.98
	6	1	0.96			4	1
		2	1	0.8		20	5
	5	2	0.95				6

**Table 6:** Continue.

3	1			7	1		
		4	1			8	1
	10	2	0.99		30	4	0.9
		3	1			5	0.98
		4	1			6	0.99
						7	1
					40	4	0.98
						5	0.99
						6	1

**Conclusion:**

Based on the sample of cases that have been discussed, it can be concluded that by using this model, obtained by the proportion of retail demand fulfillment immediately from stock on hand an average of more than 75%. Then the proportion of retail demand is met by emergency lateral transshipments average no more than 20%. Thus proved that the emergency lateral transshipments to increase the level of retail demand fulfillment. This can be seen on the results of calculations which was initially 75%, after using emergency lateral transshipments retail demand fulfillment rate rose by 20%.

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